

Abstract: The well known Planck Constants Length, Time, Mass and Force will be extended by the Planck Acceleration. This paper shows how to solve Einstein's Field Equations evaluated in Robertson Walker metric, within the Planck Era context.

1 Planck Constants

$$\text{Planck Length } \Delta x = \sqrt{\frac{Gh}{c^3}}$$

$$\text{Planck Time } \Delta t = \sqrt{\frac{Gh}{c^5}}$$

$$\text{Planck Mass } \Delta m = \sqrt{\frac{hc}{G}}$$

$$\text{Planck Force } \Delta F = \frac{c^4}{G}$$

$$\text{Planck Acceleration } \Delta a = \frac{c}{\Delta t} = \sqrt{\frac{c^7}{Gh}}$$

2 General Relativity

Einstein's original field equations are (see [1]):

$$R_{ik} - \frac{1}{2}g_{ik}R - \Lambda g_{ik} = \frac{8\pi G}{c^4}T_{ik} \quad (2.0)$$

Einstein's field equations expressed in Friedmann Robertson Walker metric will give the following two equations (curvature parameter k takes on values +1, 0 or -1):

FRW Equation (I)

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{R^2} \quad (2.1)$$

FRW Equation (II)

$$\frac{\ddot{R}}{R} = \frac{\Lambda c^2}{3} - \frac{4\pi G}{3} \left(\rho + \frac{3P}{c^2} \right) \quad (2.2)$$

3 Einstein's Field Equations in Planck Era

We assume cosmological term $\Lambda = 0$, $k = 0$, $R = \Delta x$, $\dot{R} = \frac{\Delta x}{\Delta t} = c$, $\ddot{R} = -\Delta a$ and the field equations as given in (2.1) and (2.2):

Planck Era equation (I)

$$3\frac{c^2}{\Delta x^2} = 8\pi G\rho \quad (3.1)$$

Planck Era equation (II)

$$-3\frac{\Delta a}{\Delta x} = -4\pi G\left(\rho + \frac{3P}{c^2}\right) \quad (3.2)$$

Within an ideal photon gas $P = \frac{\rho c^2}{3}$ will equation (3.2) change to:

$$-3\frac{\Delta a}{\Delta x} = -8\pi G\rho \quad (3.3)$$

In Planck Time we can assume:

$$\frac{c^2}{\Delta x^2} = \frac{\Delta a}{\Delta x} = \frac{1}{\Delta t^2} \quad (3.4)$$

The equations (3.1) and (3.3) are equal and we receive:

$$\frac{3}{\Delta t^2} = 8\pi G\rho \quad (3.5)$$

From equation (3.5) we can calculate the density ρ :

$$\rho = \frac{3}{8\pi G\Delta t^2} \quad (3.6)$$

The pressure term is given by:

$$p = \frac{\rho c^2}{3} = \frac{c^2}{8\pi G\Delta t^2} \quad (3.7)$$

4 Calculation of the total force

The total force given by the pressure term is given by:

$$F = pA = \frac{c^2}{8\pi G\Delta t^2} 4\pi\Delta x^2 = \frac{\Delta F}{2} = \frac{c^4}{2G} \quad (3.8)$$

5 Planck Era proof by Thermodynamics

The total energy is given by $E = (\rho c^2 + 3p)V$ and with (3.6) and (3.7):

$$E = \left(\frac{3c^2}{8\pi G\Delta t^2} + \frac{3c^2}{8\pi G\Delta t^2}\right)V = \left(\frac{3c^2}{4\pi G\Delta t^2}\right)V \quad (3.9)$$

With $V = \frac{4}{3}\Delta x^3\pi$ we calculate:

$$E = \frac{c^2\Delta x^3}{G\Delta t^2} \quad (3.10)$$

The definition of $G = \frac{\Delta x^3}{\Delta t^2\Delta m}$ gives the expected result:

$$E = \Delta mc^2 \quad (3.11)$$

6 Conclusion

1. We assumed Planck Era, curvature parameter $k = 0$ and no Cosmological term Λ and we calculated the values for density ρ , pressure p and give a thermodynamical proof of result.
2. We assumed an ideal gas with $P = \frac{\rho c^2}{3}$ and saw that the half mass of the universe is expressed as mass density ρ and the other half is given by the pressure term.
3. Within Planck's Mechanics there are two term which are interesting. First the mass production term $\frac{\Delta m}{\Delta t} = \frac{c^3}{G}$ and the existance of Black Holes $\frac{\Delta m}{\Delta x} = \frac{c^2}{G}$. The FRW Equation (I) $\frac{\dot{R}^2}{R^2} = \frac{1}{\Delta t^2}$ and the FRW Equation (II) $\frac{\ddot{R}}{R} = -\frac{1}{\Delta t^2}$.

7 References

1. A.Einstein, Sitz. Preuss. Akad. d. Wiss. Phys.-Math 142 (1917)
2. V.Sahni, The Case for a Positive Cosmological Λ -Term, astro-ph/9904398
3. S.M.Carroll, The Cosmological Constant, astro-ph/0004075