

# Planck Constants solves Einstein Field Equations with $k=+1$ and positive Lambda

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## Abstract

The well known Planck Constants Length, Time and Mass will be extended by the Planck Acceleration. This paper shows how to solve Einstein's Field Equations evaluated in Robertson Walker metric, within the Planck Era context.

## 1 Planck Constants

$$\text{Planck Length } \Delta x = \sqrt{\frac{Gh}{c^3}}$$

$$\text{Planck Time } \Delta t = \sqrt{\frac{Gh}{c^5}}$$

$$\text{Planck Mass } \Delta m = \sqrt{\frac{hc}{G}}$$

$$\text{Planck Acceleration } \Delta a = \frac{c}{\Delta t} = \sqrt{\frac{c^7}{Gh}}$$

## 2 General Relativity

Einstein's original field equations are (see [1]):

$$R_{ik} - \frac{1}{2}g_{ik}R - \Lambda g_{ik} = \frac{8\pi G}{c^4}T_{ik} \quad (2.0)$$

Einstein's field equations expressed in Friedmann Robertson Walker metric will give the following two equations (curvature parameter  $k$  takes on values  $+1$ ,  $0$  or  $-1$ ):

FRW Equation (I)

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{R^2} \quad (2.1)$$

FRW Equation (II)

$$\frac{\ddot{R}}{R} = \frac{\Lambda c^2}{3} - \frac{4\pi G}{3} \left( \rho + \frac{3P}{c^2} \right) \quad (2.2)$$

### 3 Einstein's Field Equations in Planck Era

We assume a cosmological term  $\Lambda$ ,  $k = +1$ ,  $R = \Delta x$ ,  $\dot{R} = \frac{\Delta x}{\Delta t} = c$ ,  $\ddot{R} = -\Delta a$  and the field equations as given in (2.1) and (2.2):

Planck Era equation (I)

$$3 \frac{c^2}{\Delta x^2} = 8\pi G\rho + \Lambda c^2 - 3 \frac{c^2}{\Delta x^2} \quad (3.1)$$

Planck Era equation (II)

$$-3 \frac{\Delta a}{\Delta x} = \Lambda c^2 - 4\pi G \left( \rho + \frac{3P}{c^2} \right) \quad (3.2)$$

Within an ideal photon gas  $P = \frac{\rho c^2}{3}$  and equation (3.2) will change to:

$$-3 \frac{\Delta a}{\Delta x} = \Lambda c^2 - 8\pi G\rho \quad (3.3)$$

In Planck Time we can assume:

$$\frac{c^2}{\Delta x^2} = \frac{\Delta a}{\Delta x} = \frac{1}{\Delta t^2} \quad (3.4)$$

If we add the equations (3.1) and (3.3) we receive:

$$0 = 2\Lambda c^2 - \frac{3}{\Delta t^2} \quad (3.5)$$

From equation (3.5) we can calculate the cosmological term  $\Lambda$ :

$$\Lambda = \frac{3}{2\Delta x^2} \quad (3.6)$$

We insert  $\Lambda$  from equation (3.6) into equation (3.1) as follows:

$$\frac{3}{\Delta t^2} = 8\pi G\rho + \frac{3}{2\Delta x^2}c^2 - \frac{3}{\Delta t^2} \quad (3.7)$$

From equation (3.7) with  $c^2 = \frac{\Delta x^2}{\Delta t^2}$  we can calculate the density  $\rho$ :

$$\rho = \frac{9}{16\pi G\Delta t^2} \quad (3.8)$$

In our calculation within Planck Era we receive:

$$\Omega_\Lambda = \frac{\Lambda c^2}{3\Delta t^{-2}} = \frac{1}{2}$$

The vacuum density is given by:

$$\rho_{vac} = \Omega_\Lambda \rho_c = \frac{1}{2} \frac{3}{8\pi G\Delta t^2} = \frac{3}{16\pi G\Delta t^2} \quad (3.9)$$

## 4 Planck Era proof by Total Mass

The total energy is given by  $E = (\epsilon + \epsilon_{vac})V$  and with (3.8) and (3.9):

$$E = \left(\frac{9}{16\pi G\Delta t^2} + \frac{3}{16\pi G\Delta t^2}\right)c^2V = \left(\frac{3}{4\pi G\Delta t^2}\right)c^2V \quad (3.10)$$

With  $V = \frac{4}{3}\Delta x^3\pi$  we calculate:

$$E = \frac{c^2\Delta x^3}{G\Delta t^2} \quad (3.11)$$

The definition of  $G = \frac{\Delta x^3}{\Delta t^2\Delta m}$  gives the expected result:

$$E = \Delta mc^2 \quad (3.12)$$

## 5 Conclusion

1. We assumed Planck Era, curvature parameter  $k = +1$  and a positive Cosmological term  $\Lambda$  and we calculated the values for density  $\rho$  and the Cosmological term  $\Lambda$ .
2. The observations of high shift supernovae indicates a actual Cosmological Omega  $\Omega_\Lambda = \frac{\Lambda c^2}{3H^2} \approx 0.7$  (see [2]). The Hubble Constant  $H$  takes values between 50 and 100  $\frac{km}{sMpc}$ . From our calculation within Planck Era we receive:

$$\Omega_\Lambda = \frac{1}{2}$$

## 6 References

1. A.Einstein, Sitz. Preuss. Akad. d. Wiss. Phys.-Math 142 (1917)
2. V.Sahni, The Case for a Positive Cosmological  $\Lambda$ -Term, astro-ph/9904398
3. S.M.Carroll, The Cosmological Constant, astro-ph/0004075